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INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **39** (2006) 4431–4439 doi:10.1088/0305-4470/39/17/S18

Equation of state of weakly nonideal plasmas and electroneutrality condition

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Received 15 September 2005, in final form 16 November 2005
Published 7 April 2006
Online at stacks.iop.org/JPhysA/39/4431

Abstract

Derivation of weakly nonideal hydrogen plasma EOS and detailed results on partial contributions to plasma pressure for the Sun's interior are presented. The impetus for this work was the demand for high accuracy of the EOS of the solar plasma in relation to the problems of modern helioseismology, accuracy sufficient for reproducing the velocity of sound on the Sun from optical measurements with errors not exceeding 10^{-4}. In our computations the relativistic corrections, degeneracy of electrons, radiation pressure in plasma, the Coulomb interaction in the Debye–Hückel approximation together with diffraction and exchange corrections and the contribution of bound and scattering states are taken into account. The analysis of the electrical neutrality condition in terms of activities and concentrations is presented. It is shown how to modify the relation between activities and concentrations for removing divergences of the Hartree contribution, representing the first order correction due to the Coulomb interaction in plasma. For the conditions of the Sun trajectory it is shown that the widely used practice of ignoring the neutrality condition in terms of activities, taking the Hartree contribution into account, gives a maximal error for plasma pressure of the order of 10^{-5}.

PACS numbers: 51.30.+i, 52.25.Kn, 52.27.Gr

1. Introduction

Helioseismology opens a unique possibility of checking within an accuracy of better than 10^{-4} the equation of state (EOS) of weakly nonideal plasmas due to the inversion of local sound velocity from optical observations [3]. The comparison of different theoretical models with experiment permits us to check the existing ways to account for bound and scattering states contributions, which are presented in physical literature for the second virial coefficient (SVC) [10, 15].

The contribution of bound states is described by the Planck–Larkin (P–L) partition function [7, 14]. An expression different from the P–L equation for the bound states contribution was published in [16]. This expression was later confirmed in [18, 19, 21].

In our work a weakly nonideal hydrogen plasma EOS is derived. We present detailed results on partial contributions to plasma pressure for conditions close to the Sun's interior [3]. The generalization of the partition function is also presented taking into account broadening of the atomic states. Analysis of the electrical neutrality condition for the Sun's interior is also performed.

2. Thermodynamic perturbation theory

According to [18] we can calculate the plasma pressure P using the corrections to the pressure P_0 of an ideal gas [13] that consists of electrons and protons:

$$P = P_0 + P_{\mathrm{H}} + P_{\mathrm{exch}} + P_{\mathrm{D-H}} + \delta P, \tag{1}$$

where we include the following corrections, discussed later in this section: P_{H} is the Hartree correction, P_{exch} is due to the electron–electron exchange interaction, $P_{\mathrm{D-H}}$ is the plasma Coulomb interaction correction in the Debye–Hückel approximation and the higher order correction δP, which takes into account contribution from the ladder diagrams, considered in sections 3 and 5.

We shall consider protons as non-degenerate particles, their ideal gas concentration, known as activity ζ_{p}, is connected in a grand canonical ensemble with their chemical potential μ_{p} and temperature T ($\beta = T^{-1}$):

$$\zeta_{\mathrm{p}} = 2\,\lambda_{\mathrm{p}}^{-3}\,\mathrm{e}^{\beta\mu_{\mathrm{p}}}, \tag{2}$$

$\lambda_{\mathrm{p}} = \sqrt{\frac{2\pi\hbar^2}{m_{\mathrm{p}}T}}$ is the thermal de Broglie wavelength. Electrons may be degenerate (at the Sun's centre $n_{\mathrm{e}}\,\lambda_{\mathrm{e}}^3 \approx 0.6$, where $\lambda_{\mathrm{e}} = \sqrt{\frac{2\pi\hbar^2}{m_{\mathrm{e}}T}}$), so we shall express their activity ζ_{e} via ideal gas concentration [13], taking into account relativistic correction up to the first order [18] according to values of the Sun's temperature. For electron gas pressure we also use an expression with the first order relativistic correction [18].

For this approximation the electroneutrality condition is written in the general form for a multicomponent plasma (where z_k is the charge of particles of kind k):

$$\sum_k z_k\zeta_k = \zeta_{\mathrm{e}}. \tag{3}$$

The Hartree correction has the following representation [7, 13, 20] for the Helmholtz thermodynamic potential $\Omega = -PV$ (V is the system volume):

$$\frac{\delta\Omega_{\mathrm{H}}}{V} = \widetilde{V}(0)\cdot\left(\zeta_{\mathrm{e}} - \sum_k z_k\zeta_k\right)^2 = -P_{\mathrm{H}}(\varkappa). \tag{4}$$

Here $\widetilde{V}(0)$ is the Fourier transform of the Coulomb potential at zero transferred momentum. Using the regularization of the integral by means of $\mathrm{e}^{-\varkappa r}$, $\varkappa \to 0$ being an infinitesimal parameter, we shall obtain ($\boldsymbol{r} = (r_1, r_2, r_3)$, $|\boldsymbol{r}| = r$, $\mathrm{d}\boldsymbol{r} = \mathrm{d}r_1\,\mathrm{d}r_2\,\mathrm{d}r_3$)

$$\widetilde{V}(\boldsymbol{q}) = \lim_{\varkappa\to 0}\int \frac{e^2}{r}\,\mathrm{e}^{\mathrm{i}\boldsymbol{q}\boldsymbol{r}-\varkappa r}\mathrm{d}\boldsymbol{r} = \frac{4\pi e^2}{q^2+\varkappa^2}, \qquad \widetilde{V}(0) = 4\pi e^2\varkappa^{-2}. \tag{5}$$

It is important that only the neutrality condition in form (3) provides a finite value for the Hartree correction (4). In the next Hartree–Fock approximation we obtain the well-known convergent result [9, 11, 13] for the electron–electron exchange interaction (see below).

Next order terms in the interaction potential, called ring diagrams [4], correspond to the Debye–Hückel contribution (see, for example, [1, 5–7, 12, 14, 20])

$$\frac{\delta\Omega_{\mathrm{D-H}}}{V} = -T\frac{\varkappa_{\mathrm{D}}^3}{12\pi}. \tag{6}$$

Here $\varkappa_{\mathrm{D}}$ is the inverse Debye radius [1] (the sum over m includes ions and electrons):

$$\varkappa_{\mathrm{D}}^2 = 4\pi e^2 \sum_m z_m^2 \left(\frac{\partial n_m}{\partial \mu_m}\right)_T. \tag{7}$$

For the non-degenerate case we can obtain in the first approximation over the $\lambdabar\varkappa$ parameter [6] the so-called diffraction correction to the Debye–Hückel term:

$$\frac{\delta\Omega_{\mathrm{diff}}}{V} = \frac{\pi}{8}T\left(\frac{e^2}{T}\right)^2\left(\lambdabar_{\mathrm{ee}}\zeta_{\mathrm{e}}^2 + 2\zeta_{\mathrm{e}}\sum_k \zeta_k z_k^2 \lambdabar_{\mathrm{e}k} + \sum_{kj}\zeta_k\zeta_j z_k^2 z_j^2 \lambdabar_{kj}\right). \tag{8}$$

Here we use $\lambdabar_{kj} = \sqrt{\frac{2\pi\hbar^2}{\mu_{kj}T}}$, $\mu_{kj} = \frac{m_k m_j}{m_k + m_j}$ is the reduced mass.

We must recall that the physical concentrations are usually connected with chemical potentials by relation [13]

$$n_m = -\left(\frac{\partial(\Omega/V)}{\partial\mu_m}\right)_T. \tag{9}$$

For physical concentrations the standard electrical neutrality condition exists:

$$n_{\mathrm{e}} = \sum_k z_k n_k. \tag{10}$$

To adjust conditions (3) and (10) we shall use the following method. We determine the physical concentrations from condition (9), taking into account the bounds, following from (3). Let us find the value of n_{e} from (10), and ζ_{e} from (3).

$$n_{\mathrm{e}} + \sum_k n_k = \sum_k (z_k+1)n_k = -\beta\sum_k\left(\frac{\partial(\Omega/V)}{\partial\zeta_k}\right)_T \zeta_k. \tag{11}$$

In expression (11) all derivations and summations are performed over ionic activities only and ζ_{e} in potential Ω is expressed using (3).

The Debye–Hückel model must be improved with SVC corrections (order of Γ_{D}^2 and higher, where $\Gamma_{\mathrm{D}} = \varkappa_{\mathrm{D}}e^2/T$ is the nonideality parameter). Here we neglect such corrections just to illustrate the difference between definitions (9) and (11). Within the framework of the model, defined by equation $\Omega = \Omega_0 + \delta\Omega_{\mathrm{D-H}}$, from condition (11) it follows that

$$\zeta_k = \frac{n_k}{1+\frac{\Gamma_{\mathrm{D}}}{2}z_k}, \qquad \zeta_{\mathrm{e}} = \sum_k \frac{z_k n_k}{1+\frac{\Gamma_{\mathrm{D}}}{2}z_k} \tag{12}$$

and, obviously, (3) is valid. In contrast, in the standard theory, if the values of activities are used in (7) instead of concentrations n_k, from (9) we obtain

$$n_k = \zeta_k\left(1+\frac{\Gamma_{\mathrm{D}}}{2}z_k^2\right), \qquad n_{\mathrm{e}} = \zeta_{\mathrm{e}}\left(1+\frac{\Gamma_{\mathrm{D}}}{2}\right). \tag{13}$$

Note the difference in the power of z_k in (12) and (13). For $z_k \neq 1$ from (13), taking account of $n_{\mathrm{e}} = \sum_k z_k n_k$, we obtain violation of (3):

$$\zeta_e = \frac{n_e}{1+\frac{\Gamma_{\mathrm{D}}}{2}} = \frac{\sum_k z_k n_k}{1+\frac{\Gamma_{\mathrm{D}}}{2}} \neq \sum_k z_k\zeta_k = \sum_k \frac{z_k n_k}{1+\frac{\Gamma_{\mathrm{D}}}{2}z_k^2}. \tag{14}$$

3. Ladder approximation for SVC calculation

Let us consider the contribution δP in (1). From the Matsubara technique (see [18] and references therein) for $\delta\Omega/V = -\delta P$ we have

$$\frac{\delta\Omega_{\rm L}}{V} = \frac{2}{\beta}\sum_{i,\omega}\int_0^1 \frac{{\rm d}\lambda}{2\lambda}\int\frac{{\rm d}\boldsymbol{p}}{(2\pi)^3}G_i(\hbar\boldsymbol{p},\omega)\Sigma_i(\hbar\boldsymbol{p},\omega). \tag{15}$$

Here the subscript 'L' stands for 'ladder', integration over parameter λ corresponds to charge integration $e^2 \mapsto e^2\lambda$. Summation is performed over particle kinds i and energies ω (or p_4)—for fermions $\omega = \pi T(2n+1)$, $\hbar\boldsymbol{p}$ is the particle momentum, $G_i(\hbar\boldsymbol{p},\omega)$ is Green's function of a particle in the Matsubara technique. The self-energy operator $\Sigma_i(\hbar\boldsymbol{p},\omega)$ can be expressed via two-particle vertex part Γ_{ij}, obtained in the ladder approximation [9]:

$$\Sigma_i(\mathfrak{p}) = \frac{2}{\beta}\sum_{j,k_4}\int\frac{{\rm d}\boldsymbol{k}}{(2\pi)^3}G_j(\mathfrak{k})\Gamma_{ij}\left(\frac{m_j\mathfrak{p}-m_i\mathfrak{k}}{m_i+m_j},\frac{m_j\mathfrak{p}-m_i\mathfrak{k}}{m_i+m_j};\mathfrak{p}+\mathfrak{k}\right). \tag{16}$$

For example, for electron–proton interaction $m_i = m_{\rm e}$, $m_j = m_{\rm p}$, $\mathfrak{p} = (\hbar\boldsymbol{p}, p_4) \equiv (\hbar\boldsymbol{p},\omega)$ is the 4-vector for electrons and $\mathfrak{k} = (\hbar\boldsymbol{k}, k_4)$ for protons, respectively. The quantity $\Gamma_{ij}(\mathfrak{q},\mathfrak{q}';\mathfrak{P})$ may be written in the following form:

$$\Gamma_{\rm ep}(\mathfrak{q},\mathfrak{q}';\mathfrak{P}) = \widetilde{V}_{\rm ep}(\boldsymbol{q}-\boldsymbol{q}') + (2\pi)^3\sum_n\frac{\widetilde{\Psi}_n(\boldsymbol{q})\widetilde{\Psi}_n^*(\boldsymbol{q}')\left(E_n-\frac{\hbar^2q^2}{2\mu}\right)\left(E_n-\frac{\hbar^2q'^2}{2\mu}\right)}{{\rm i}P_4-\frac{\hbar^2P^2}{2M}-E_n+\mu_{\rm e}+\mu_{\rm p}}, \tag{17}$$

where $\mathfrak{q}$, $\mathfrak{q}'$ are the relative motion momenta before and after scattering, $\mathfrak{P} = \mathfrak{p}+\mathfrak{k} = (\hbar\boldsymbol{P}, P_4)$ is the 4-vector for total momentum, $\mu = \mu_{\rm ep}$ is the reduced mass. From (15)–(17) we derive $\delta\Omega$ in a form using the electrical neutrality condition in terms of activities (3):

$$\frac{\delta\Omega_{\rm L}}{V} = \sum_{i,j}\zeta_i\zeta_j\,\lambda\!\!\!^{-}{}_{ij}^3\int_0^1\frac{{\rm d}\lambda}{2\lambda}\int\frac{{\rm d}\boldsymbol{q}}{(2\pi)^3}\sum_n\left(E_n-\frac{\hbar^2q^2}{2\mu}\right)|\widetilde{\Psi}_n(\boldsymbol{q})|^2({\rm e}^{-\beta E_n}-{\rm e}^{-\beta\varepsilon_q}). \tag{18}$$

In (18) for 'e–p' interaction it is necessary to sum over discrete spectrum (bound states) as well as to integrate over scattering states, described by index $\boldsymbol{k}$. For 'e–e' and 'p–p' interaction only the last action is not unreasonable. Such an approach permits independent verification of the expressions for SVC published in the literature [7, 12].

4. Contribution to the SVC from bound states

To get the contribution of bound states to the SVC as in [19] we use the exact Fock [8] result for the wave functions of a non-relativistic hydrogen atom in the momentum representation

$$\frac{1}{(2\pi)^3}\sum_{l,m}|\widetilde{\Psi}_{n,l,m}(\boldsymbol{q})|^2 = \frac{8}{\pi^2a_0^5n^3\left(q^2+p_n^2\right)^4}. \tag{19}$$

Here $p_n = (a_0n)^{-1}$, $a_0 = \frac{\hbar^2}{\mu e^2\lambda}$ is the Bohr 'radius' for current charge $e^2\lambda$. Taking into account the fact that $E_n \equiv -\frac{\hbar^2p_n^2}{2\mu}$ and using (19) the part of expression (18) corresponding to bound states may be written in the form:

$$\frac{\delta\Omega_{\rm SRM}^{\rm BS}}{V} = -\zeta_{\rm e}\zeta_{\rm p}T\,\lambda\!\!\!^{-}{}_{\rm ep}^3\,\Sigma_{\rm SRM}^{\rm BS} = -\zeta_{\rm e}\zeta_{\rm p}T\,\lambda\!\!\!^{-}{}_{\rm ep}^3\sum_{n=1}^{\infty}n^2\,{\rm e}^{\chi_n}F(\chi_n). \tag{20}$$

Here $\chi_n = \beta\mathrm{Ry}/n^2$ and

$$F(\chi) = 1 - \mathrm{e}^{-\chi}\left(4 - \frac{6}{\sqrt{\pi}}\chi^{1/2} + \frac{4}{\sqrt{\pi}}\chi^{3/2}\right) + \frac{\Gamma\left(\frac{1}{2},\chi\right)}{\sqrt{\pi}}(3 - 4\chi + 4\chi^2). \tag{21}$$

As was shown in [19], asymptotically for $n \gg 1$ this expression is four times greater than the similar expression in the Planck–Larkin partition function:

$$F_{\mathrm{P-L}}(\chi) = 1 - \mathrm{e}^{-\chi} - \chi\,\mathrm{e}^{-\chi}, \qquad \Sigma^{\mathrm{BS}}_{\mathrm{P-L}} = \sum_{k=2}^{\infty}\frac{\zeta(2k-2)\alpha^{2k}}{\Gamma(k+1)}, \tag{22}$$

where $\zeta(k)$ is the Riemann ζ-function. The Planck–Larkin expression contains some contribution from scattering states, while (20)–(21) really represent the bound states contribution only.

The bound states contribution may be written as

$$\delta\Omega^{\mathrm{BS}}V^{-1} = -\zeta_{\mathrm{e}}\zeta_{\mathrm{p}}T\,\lambda^3_{\mathrm{ep}}\Sigma^{\mathrm{BS}}. \tag{23}$$

Note that expressions like (18) are obtained using the Keldysh technique [18]:

$$\delta P = \sum_a(2S_a+1)\int_0^1\frac{\mathrm{d}\lambda}{2\lambda}\int\frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3}\int\frac{\mathrm{d}\omega\,\mathrm{d}\omega'}{(2\pi)^2}\frac{\Sigma^{>}_a(\hbar\boldsymbol{p},\omega)\,G^{<}_a(\hbar\boldsymbol{p},\omega')}{\omega-\omega'}(1-\mathrm{e}^{-\beta(\omega-\omega')}). \tag{24}$$

For the general form of the bound states contribution (23) from (24) using the broadening of Green's functions in the media we can obtain

$$\Sigma^{\mathrm{BS}} = \beta\int_0^1\frac{\mathrm{d}\lambda}{\lambda}\int\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\int\mathrm{d}\omega\sum_{n=1}^{\infty}(E_n-\varepsilon_q)^2|\widetilde{\Psi}_n(q)|^2\frac{\mathrm{e}^{-\beta\omega}-\mathrm{e}^{-\beta\varepsilon_q}}{\varepsilon_q-\omega}a_n(\omega-E_n), \tag{25}$$

where $\boldsymbol{q}$ is the wave-vector of relative motion of the particles, $q = |\boldsymbol{q}|$, $a_n(\omega)$ is the profile of the atomic state n, broadened by ions and electrons [17]. If we neglect the broadening effects and use the δ-function instead of the profile $a_n(\omega)$ in (25), we obtain (20)–(21).

5. Contribution to the SVC from scattering states

To calculate the contribution of the continual spectrum states to expressions like (18) one needs to evaluate Fourier components of the wave functions that describe mutual scattering of charged particles. It is convenient to use the system of Coulomb wave functions as a sum over orbital moments [2]. In the momentum representation we have

$$\widetilde{\Psi}_k(q) = (2\pi)^{-3/2}\exp(\pi\tilde{\xi}/2)\Gamma(1-\mathrm{i}\tilde{\xi})J_\varkappa. \tag{26}$$

$$J_\varkappa = \left(\frac{2\pi(1-\mathrm{i}\tilde{\xi})\varkappa}{\left(\frac{(\boldsymbol{q}-\boldsymbol{k})^2}{2}+\frac{\varkappa^2}{2}\right)^2} + \frac{2\pi\tilde{\xi}(k+\mathrm{i}\varkappa)}{\left(\frac{(\boldsymbol{q}-\boldsymbol{k})^2}{2}+\frac{\varkappa^2}{2}\right)\left(\frac{q^2-k^2+\varkappa^2}{2}-\mathrm{i}k\varkappa\right)}\right) \times\exp\left(\mathrm{i}\tilde{\xi}\ln\frac{(\boldsymbol{q}-\boldsymbol{k})^2+\varkappa^2}{q^2-k^2+\varkappa^2-2\mathrm{i}k\varkappa}\right). \tag{27}$$

Here $\tilde{\xi} = (a_0k)^{-1} = \frac{\mu e^2\lambda}{\hbar^2 k}$. The first term in (27) is similar to the regularized 3D δ-function with the use of parameter $\varkappa$, $\varkappa \to 0$ (compare with (5)). Similar expressions can also be obtained for 'e–e' interaction by changing $m_{\mathrm{p}} \to m_{\mathrm{e}}$.

Taking into account subtraction in the factor $(\mathrm{e}^{-\beta E_k} - \mathrm{e}^{-\beta\varepsilon_q})$ in (18), for example, for 'e–p' interaction we can transform expression (18) for the continual spectrum of scattering states (SS) to

$$\frac{\delta\Omega^{\mathrm{SS}}_{\mathrm{ep}}}{V} = -\frac{\lambda\!\!\!^{-}_{\mathrm{ep}}}{\pi}\zeta_{\mathrm{e}}\zeta_{\mathrm{p}}T\left(\frac{e^2}{T}\right)^2\int_0^1\lambda\,\mathrm{d}\lambda\int_0^\infty\int_0^\infty \mathrm{d}x\,\mathrm{d}y\,\frac{\pi\tilde{\xi}_{\mathrm{ep}}}{\mathrm{sh}\,\pi\tilde{\xi}_{\mathrm{ep}}}\exp(\pi\tilde{\xi}_{\mathrm{ep}})\frac{\mathrm{e}^{-y}-\mathrm{e}^{-x}}{x-y}$$
$$\times\left(\frac{1}{(\sqrt{x}-\sqrt{y})^2+\tilde{\eta}^2_{\mathrm{ep}}}-\frac{1}{(\sqrt{x}+\sqrt{y})^2+\tilde{\eta}^2_{\mathrm{ep}}}\right)$$
$$\times\exp\left[-2\tilde{\xi}_{\mathrm{ep}}\,\mathrm{Im}\ln\left(x-y+\tilde{\eta}^2_{\mathrm{ep}}+\mathrm{i}2\sqrt{y\tilde{\eta}^2_{\mathrm{ep}}}\right)\right], \tag{28}$$

where $\tilde{\xi}_{\mathrm{ep}} = \frac{\lambda\alpha_{\mathrm{ep}}}{\sqrt{y}}$, $\alpha_{\mathrm{ep}} = \sqrt{\mathrm{Ry}/T}$, $\tilde{\eta}^2_{\mathrm{ep}} = \frac{\hbar^2\varkappa^2}{8\mu T}$.

3D numerical integration provides the following asymptotic as $\varkappa \to 0$ $(a = \mathrm{e}, \mathrm{p})$

$$\frac{\delta\Omega^{\mathrm{SS}}_{aa}}{V} - \frac{\delta\Omega^{(2)}_{aa}}{V} \to -\zeta_a^2\left\{\frac{e^6}{T^2}\left[\frac{\pi}{3}\ln\left(\frac{\varkappa\lambda\!\!\!^{-}_{aa}}{2\sqrt{\pi}}\right)-\frac{\pi}{6}(1-C)\right]+\frac{\lambda\!\!\!^{-3}_{aa}T}{2}\Sigma_{\mathrm{Q}}\left(-\frac{\alpha_a}{2}\right)\right\}, \tag{29}$$

where $\delta\Omega^{(2)}_{aa}$ is a part of a ladder with two steps, already included in the Debye–Hückel approximation, $\alpha_a = \sqrt{\frac{m_a e^4}{\hbar^2 T}}$:

$$\frac{\delta\Omega^{\mathrm{SS}}_{\mathrm{ep}}}{V} - \frac{\delta\Omega^{(2)}_{\mathrm{ep}}}{V} \to 2\zeta_{\mathrm{e}}\zeta_{\mathrm{p}}\left\{\frac{e^6}{T^2}\left[\frac{\pi}{3}\ln\left(\frac{\varkappa\lambda\!\!\!^{-}_{\mathrm{ep}}}{2\sqrt{\pi}}\right)-\frac{\pi}{6}(1-C)\right]+\frac{\lambda\!\!\!^{-3}_{\mathrm{ep}}T}{2}\left[\Sigma^{\mathrm{BS}}_{\mathrm{SRM}}-\Sigma_{\mathrm{Q}}(\alpha_{\mathrm{ep}})\right]\right\},$$
$$\Sigma_{\mathrm{Q}}(\alpha) = \frac{1}{2}\sum_{n=4}^{\infty}\frac{\zeta(n-2)}{\Gamma\left(\frac{n}{2}+1\right)}\alpha^n = -\left(\ln|2\alpha|+\frac{3C}{2}-\frac{4}{3}\right)\frac{2\alpha^3}{3\sqrt{\pi}}+o(|\alpha|^3) \quad \text{as} \quad \alpha\to-\infty. \tag{30}$$

In [7] the second item inside brackets in (30) contains an excess term with ln 3 in parentheses $(-C - 2\ln 3 + 1)$ as a mistake.

Summation of expressions like (30) for 'p–p', 'e–e' and 'e–p' interactions taking into account that $\zeta_{\mathrm{e}} = \zeta_{\mathrm{p}}$ will result in an expression that is independent of $\varkappa$ for a classical part of SVC:

$$\frac{\delta\Omega^{\mathrm{cl}}}{V} = \zeta_{\mathrm{e}}^2 T\left(\frac{e^2}{T}\right)^3\cdot\frac{\pi}{6}\ln\frac{m_{\mathrm{p}}}{4m_{\mathrm{e}}}. \tag{31}$$

For exchange contribution $\delta\Omega^{\mathrm{exch}}_{\mathrm{ee}}$ we obtain a convergent expression

$$\frac{\delta\Omega^{\mathrm{exch}}_{\mathrm{ee}}}{V} = \frac{1}{8\sqrt{\pi}}\zeta_{\mathrm{e}}^2\,\lambda\!\!\!^{-3}_{\mathrm{ee}}T\,E(\alpha_{\mathrm{ee}}). \tag{32}$$

For $E(\alpha_{\mathrm{ee}})$ one can get an explicit expression $(\alpha_{\mathrm{ee}} = -\alpha_{\mathrm{e}}/2)$

$$E(\alpha) = \alpha + \sqrt{\pi}\ln 2\cdot\alpha^2 + \frac{\pi^2}{9}\alpha^3 + \sum_{n=4}^{\infty}\frac{\sqrt{\pi}(1-2^{2-n})}{\Gamma\left(\frac{n}{2}+1\right)}\zeta(n-1)\alpha^n. \tag{33}$$

Expression (33) is just the same as in [7] for exchange contribution and is confirmed by numerical integration.

6. Weakly nonideal hydrogen plasmas EOS along the Sun's trajectory

Consider the total contribution of ideal gas pressure and expressions (6), (8), (23), (29), (30), (31), (32) to describe the EOS of weakly nonideal hydrogen plasmas in application to

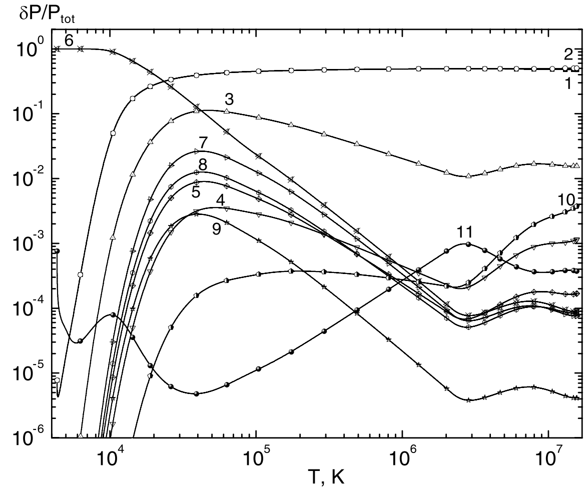

Figure 1. Absolute values of specific partial contributions ($\delta P/P_{\text{tot}}$) to total pressure as functions of temperature 1—$P_{0\text{p}}$; 2—$P_{0\text{e}}$; 3—$P_{\text{D-H}}$; 4—$\delta\Omega_{\text{diff}}/V$; 5—$\delta\Omega^{\text{cl}}/V$; 6— $-\delta\Omega^{\text{BS}}_{\text{SRM}}/V$; 7—$\delta\Omega^{q}_{\text{ep}}/V$; 8— $-\delta\Omega^{q}_{\text{pp}}/V$; 9— $-\delta\Omega^{q}_{\text{ee}}/V$; 10— $-\delta\Omega^{\text{exch}}_{\text{ee}}/V$; 11—$P_{\text{R}}$.

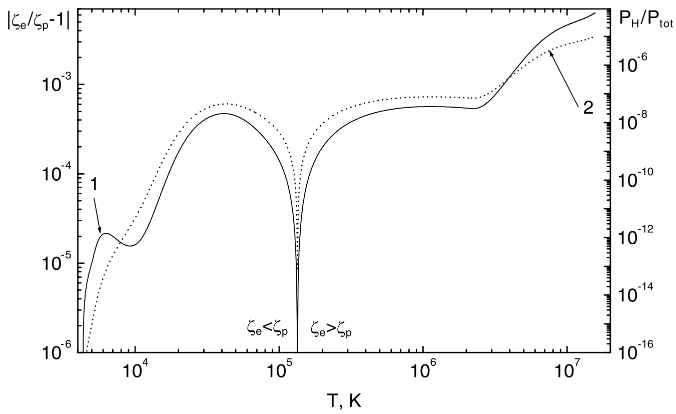

Figure 2. Electroneutrality analysis for the Sun's interior, the functions of temperature: 1 (left axis)—deviation from neutrality in terms of activity $|\zeta_{\text{e}}/\zeta_{\text{p}} - 1|$ using standard definition in concentrations defined as $n_k = -\frac{\partial(\Omega/V)}{\partial\mu_k}$; 2 (right axis)—total estimation of Hartree term $P_{\text{H}}(\varkappa_{\text{D}})/P_{\text{tot}}$ of contribution due to deviation from condition (3).

helioseismology problems. When considering the plasma EOS (value of pressure $P(\rho, T)$ and other thermodynamic functions) the contribution of equilibrium thermal radiation in plasma should be added [18].

We calculated a weakly nonideal hydrogen plasma EOS that is a function of total pressure $P(T)$ along the S-model of the Sun's interior distribution [3].

Figure 1 shows the temperature dependence of partial contributions to total pressure $\delta P/P_{\text{tot}}$. It is evident that many corrections to the SVC are significant, if high accuracy of the inversion in helioseismology problems is taken into account.

Figure 2 presents the results of a electroneutrality analysis for the Sun's interior. The difference between ζ_{e} and ζ_{p}, which is shown in figure 2, appears if we neglect the electroneutrality condition in terms of activities and find ζ_m from the 'standard' definition of concentration (9). In such an incorrect calculation of activities we used the EOS of weakly nonideal hydrogen plasmas with non-zero Hartree correction (4) $P_{\mathrm{H}}(\varkappa_{\mathrm{D}})$.

7. Conclusion

The new results presented in this paper may be underlined as follows:

- new definition of electroneutrality condition in terms of activities and its conjugation with the same condition in terms of concentrations,
- new correct expression for bound states and scattering states contribution, different from those well known from the textbooks [7, 12],
- numerical estimations using corrected EOS for different contributions to the pressure along the Sun's trajectory,
- estimation of the influence of the new electroneutrality condition for the Sun's interior.

Successive account of the electroneutrality condition in terms of activities equality eliminates divergence in the Hartree term, in logarithmic contributions, proportional to e^6, and also in the contribution of bound states and scattering states. Fortunately, the account of neutrality in the traditional form (without requirement of equal activities) leads to a relatively small error from the Hartree term contribution (see figure 2)

$$P_{\mathrm{H}}(\varkappa_{\mathrm{D}}) \sim 10^{-5} P_{\mathrm{tot}}.$$

In our opinion the account of broadening effects (see (25) and [19]) is more promising for spectral lines description as well as for weakly nonideal dense plasma thermodynamics. In principle, this approach joins problems of radiational gasdynamics and collisional–radiative kinetics, where 'atoms' are represented in a different way for calculation of pressure and radiation [19].

Acknowledgments

The authors are grateful to S V Ayukov, V A Baturin, V E Fortov, V K Gryaznov, I L Iosilevskii, T Kato and R More for series of fruitful discussions and to M G Gladush for help. This work was supported by RFBR grant no 04-02-16775-a and the President of the Russian Federation grant NSh-1257.2003.2.

References

[1] Abrikosov A A, Gorkov L P and Dzyaloshinskii I E 1977 *Methods of Quantum Field Theory in Statistical Physics* transl. and ed R A Silverman (New York: Dover)

[2] Akhiezer A I and Berestetskii V B 1965 *Quantum Electrodynamics* (New York: Interscience)

[3] Christensen-Dalsgaard J, Däppen W, Dziembowski W A and Guzik J A 2000 An introduction to helioseismology *Variable Stars as Important Astrophysical Tools: Proc. NATO Advanced Study Institute (Cesme, Turkey, 30 August–11 September 1998)* ed C Ibanoğlu (Dordrecht: Kluwer) pp 59–167 http://bigcat.ifa.au.dk/~jcd/solar_models/cptrho.l5bi.d.15c

[4] DeWitt H E 1962 Evaluation of the quantum-mechanical ring sum with Boltzmann statistics *J. Math. Phys.* **3** 1216–28

[5] Ebeling W, Hoffmann H J and Kelbg G 1967 Quantenstatistik des Hochtemperaturplasmas in thermodynamischen Gleichgewicht *Beitr. Plasmaphys.* **7** 233–48

[6] Ebeling W, Kelbg G and Rohde K 1968 Binäre Slatersummen und Verteilungsfunktionen für quantenstatistische Systeme mit Coulomb-Wechselwirkung I *Ann. Phys., Lpz.* **21** 235–43

[7] Ebeling W, Kraeft W-D and Kremp D 1976 *Theory of Bound States and Ionisation Equilibrium in Plasmas and Solids* (Berlin: Akademie)
[8] Fock V A 1935 Zur Theorie des Wasserstoffatoms *Z. Phys.* **98** 145–54
[9] Galitskii V M 1958 The energy spectrum of a non-ideal Fermi gas *Zh. Eksp. Teor. Fiz.* **34** 151–62
Galitskii V M 1958 *Sov. Phys.—JETP* **7** 104 (Engl. transl.)
[10] Hummer D G and Mihalas D 1988 The equation of state for stellar envelopes: I. An occupation probability formalism for the truncation of internal partition functions *Astrophys. J.* **331** 794–814
[11] Kadanoff L and Baym G 1962 *Quantum Statistical Mechanics* (New York: Benjamin)
[12] Kraeft W-D, Kremp D, Ebeling W and Röpke G 1986 *Quantum Statistics of Charged Particle Systems* (New York: Plenum)
[13] Landau L D and Lifshitz E M 1980 *Statistical Physics: Part 1 (Course of Theoretical Physics* vol 5*)* (Oxford: Butterworth-Heinemann) (transl. J B Sykes and M J Kearsley)
[14] Larkin A I 1960 Thermodynamic functions of a low temperature plasma *Zh. Eksp. Teor. Fiz.* **38** 1896–8
Larkin A I 1960 *Sov. Phys.—JETP* **11** 1363 (Engl. transl.)
[15] Mihalas D, Däppen W and Hummer D G 1988 The equation of state for stellar envelopes: II. Algorithm and selected results *Astrophys. J.* **331** 815–25
[16] Norman G E and Starostin A N 1970 *Teplofiz. Vys. Temp.* **8** 413–38
Norman G E and Starostin A N 1970 *High Temp.* (USSR) **8** 381 (Engl. transl.)
[17] Sobelman I I, Vainshtein L A and Yukov E A 1981 *Excitation of Atoms and Broadening of Spectral Lines* (Berlin: Springer)
[18] Starostin A N and Roerich V C 2005 A converging equation of state of a weakly nonideal hydrogen plasma without mystery *Zh. Eksp. Teor. Fiz.* **127** 186–219
Starostin A N and Roerich V C 2005 *JETP* **100** 165–98 (Engl. transl.)
[19] Starostin A N, Roerich V C and More R M 2003 How correct is the EOS of weakly nonideal hydrogen plasmas? *Contrib. Plasma Phys.* **43** 369–72
[20] Vedenov A A and Larkin A I 1959 The state equation of a plasma *Zh. Eksp. Teor. Fiz.* **36** 1133–42
Vedenov A A and Larkin A I 1959 *Sov. Phys.—JETP* **9** 806 (Engl. transl.)
[21] Vorobiev V S and Khomkin A L 1971 Canonical transformation method in thermodynamics of partially ionized plasma *Teor. Mat. Fiz.* **8** 109–18